



HÀM SỐ LƯỢNG GIÁC VÀ PHƯƠNG TRÌNH LƯỢNG GIÁC

BÀI 2. CÔNG THỨC LƯỢNG GIÁC



LÝ THUYẾT.

1. CÔNG THỨC CỘNG

$$\cos(a - b) = \cos a \cos b + \sin a \sin b$$

$$\cos(a + b) = \cos a \cos b - \sin a \sin b$$

$$\sin(a - b) = \sin a \cos b - \cos a \sin b$$

$$\sin(a + b) = \sin a \cos b + \cos a \sin b$$

$$\tan(a - b) = \frac{\tan a - \tan b}{1 + \tan a \tan b}$$

$$\tan(a + b) = \frac{\tan a + \tan b}{1 - \tan a \tan b}$$

2. CÔNG THỨC NHÂN ĐÔI

1. Công thức nhân đôi

$$\sin 2a = 2 \sin a \cos a$$

$$\cos 2a = \cos^2 a - \sin^2 a = 2 \cos^2 a - 1 = 1 - 2 \sin^2 a$$

$$\tan 2a = \frac{2 \tan a}{1 - \tan^2 a}$$

2. Công thức hạ bậc

$$\cos^2 a = \frac{1 + \cos 2a}{2}$$

$$\sin^2 a = \frac{1 - \cos 2a}{2}$$

$$\tan^2 a = \frac{1 - \cos 2a}{1 + \cos 2a}$$



3. CÔNG THỨC BIẾN ĐỔI TÍCH THÀNH TỔNG

$$\cos a \cos b = \frac{1}{2} [\cos(a-b) + \cos(a+b)]$$

$$\sin a \sin b = \frac{1}{2} [\cos(a-b) - \cos(a+b)]$$

$$\sin a \cos b = \frac{1}{2} [\sin(a-b) + \sin(a+b)].$$

4. CÔNG THỨC BIẾN ĐỔI TỔNG THÀNH TÍCH

$$\cos u + \cos v = 2 \cos \frac{u+v}{2} \cos \frac{u-v}{2}$$

$$\cos u - \cos v = -2 \sin \frac{u+v}{2} \sin \frac{u-v}{2}$$

$$\sin u + \sin v = 2 \sin \frac{u+v}{2} \cos \frac{u-v}{2}$$

$$\sin u - \sin v = 2 \cos \frac{u+v}{2} \sin \frac{u-v}{2}$$

II HỆ THỐNG BÀI TẬP TỰ LUẬN

Câu 1: Rút gọn các biểu thức sau:

a. $A = \cos\left(\frac{\pi}{2} + x\right) + \cos(2\pi - x) + \cos(3\pi + x)$

b. $B = 2 \cos x - 3 \cos(\pi - x) + 5 \sin\left(\frac{7\pi}{2} - x\right) + \cos\left(\frac{3\pi}{2} - x\right)$

c. $C = 2 \sin\left(\frac{\pi}{2} + x\right) + \sin(5\pi - x) + \sin\left(\frac{3\pi}{2} + x\right) + \cos\left(\frac{\pi}{2} + x\right)$

d. $D = \cos(5\pi - x) - \sin\left(\frac{3\pi}{2} + x\right) + \tan\left(\frac{3\pi}{2} - x\right) + \cot(3\pi - x)$

Lời giải

a. $A = \cos\left(\frac{\pi}{2} + x\right) + \cos(2\pi - x) + \cos(3\pi + x) = -\sin x + \cos x - \cos x = -\sin x$

b. $B = 2 \cos x - 3 \cos(\pi - x) + 5 \sin\left(\frac{7\pi}{2} - x\right) + \cos\left(\frac{3\pi}{2} - x\right)$

$$= 2 \cos x + 3 \cos x + 5 \sin\left(\frac{-\pi}{2} - x\right) + \cos\left(\frac{-\pi}{2} - x\right) = 5 \cos x - 5 \sin\left(\frac{\pi}{2} + x\right) + \cos\left(\frac{\pi}{2} + x\right)$$

$$= 5 \cos x - 5 \cos x - \sin x = -\sin x$$

c. $C = 2 \sin\left(\frac{\pi}{2} + x\right) + \sin(5\pi - x) + \sin\left(\frac{3\pi}{2} + x\right) + \cos\left(\frac{\pi}{2} + x\right)$

$$= 2 \cos x + \sin x + \sin\left(\frac{-\pi}{2} + x\right) - \sin x = 2 \cos x - \cos x = \cos x$$



$$\begin{aligned} \text{d. } D &= \cos(5\pi - x) - \sin\left(\frac{3\pi}{2} + x\right) + \tan\left(\frac{3\pi}{2} - x\right) + \cot(3\pi - x) \\ &= -\cos x - \sin\left(\frac{-\pi}{2} + x\right) + \tan\left(\frac{-\pi}{2} - x\right) - \cot x = -\cos x + \cos x + \cot x - \cot x = 0. \end{aligned}$$

Câu 2: Rút gọn biểu thức sau:

$$\text{a. } G = \cos(15\pi - x) + \sin\left(x - \frac{3\pi}{2}\right) - \tan\left(\frac{\pi}{2} + x\right) \cot\left(\frac{11\pi}{2} - x\right)$$

$$\text{b. } H = \sin(\pi + x) - \cos\left(\frac{\pi}{2} - x\right) + \cot(2\pi - x) + \tan\left(\frac{3\pi}{2} - x\right)$$

$$\text{c. } I = \cos(5\pi - x) - \sin\left(\frac{3\pi}{2} + x\right) + \tan\left(\frac{3\pi}{2} - x\right) + \cot(3\pi - x)$$

Lời giải

$$\begin{aligned} \text{a. } G &= \cos(15\pi - x) + \sin\left(x - \frac{3\pi}{2}\right) - \tan\left(\frac{\pi}{2} + x\right) \cot\left(\frac{11\pi}{2} - x\right) \\ &= -\cos x + \sin\left(x + \frac{\pi}{2}\right) + \cot x \cot\left(\frac{-\pi}{2} - x\right) = -\cos x + \cos x + \cot x \tan x = 1 \end{aligned}$$

$$\begin{aligned} \text{b. } H &= \sin(\pi + x) - \cos\left(\frac{\pi}{2} - x\right) + \cot(2\pi - x) + \tan\left(\frac{3\pi}{2} - x\right) \\ &= -\sin x - \sin x - \cot x + \tan\left(\frac{-\pi}{2} - x\right) = -2\sin x - \cot x + \cot x = -2\sin x \end{aligned}$$

$$\begin{aligned} \text{c. } I &= \cos(5\pi - x) - \sin\left(\frac{3\pi}{2} + x\right) + \tan\left(\frac{3\pi}{2} - x\right) + \cot(3\pi - x) \\ &= \cos(\pi - x + 4\pi) - \sin\left(-\left(\frac{\pi}{2} - x\right) + 2\pi\right) + \tan\left(\frac{\pi}{2} - x + \pi\right) + \cot(-x + 3\pi) \\ &= \cos(\pi - x) - \sin\left(-\left(\frac{\pi}{2} - x\right)\right) + \tan\left(\frac{\pi}{2} - x\right) + \cot(-x) \\ &= -\cos(x) + \sin\left(\frac{\pi}{2} - x\right) + \cot(x) - \cot(x) \\ &= -\cos(x) + \cos(x) \\ &= 0 \end{aligned}$$

Câu 3: Rút gọn biểu thức sau:

$$\text{a. } N = \sin^6(\pi + x) + \cos^6(x - \pi) - 2\sin^4(x + 2\pi) - \sin^4\left(x - \frac{3\pi}{2}\right) + \cos^2\left(x - \frac{\pi}{2}\right)$$

$$\text{b. } O = \frac{\tan\left(\frac{19\pi}{2} - x\right) \cos(36\pi - x) \sin(x - 5\pi)}{\sin\left(\frac{9\pi}{2} - x\right) \cos(x - 99\pi)}$$

$$\text{c. } P = \sin\left(x + \frac{85\pi}{2}\right) + \cos(207\pi + x) + \sin^2(33\pi + x) + \sin^2\left(x - \frac{3\pi}{2}\right)$$



Lời giải

$$\text{a. } N = \sin^6(\pi + x) + \cos^6(x - \pi) - 2\sin^4(x + 2\pi) - \sin^4\left(x - \frac{3\pi}{2}\right) + \cos^2\left(x - \frac{\pi}{2}\right)$$

$$= \sin^6 x + \cos^6 x - 2\sin^4 x - \cos^4 x + \sin^2 x$$

$$= -\sin^4 x - \sin^2 x \cos^2 x + \sin^2 x = -\sin^4 x + \sin^2 x(1 - \cos^2 x) = 0$$

$$\text{b. } O = \frac{\tan\left(\frac{19\pi}{2} - x\right) \cos(36\pi - x) \sin(x - 5\pi)}{\sin\left(\frac{9\pi}{2} - x\right) \cos(x - 99\pi)}$$

$$= \frac{\tan\left(\frac{-\pi}{2} - x\right) \cos x (-\sin x)}{\sin\left(\frac{\pi}{2} - x\right) \cos(x + \pi)} = \frac{-\cot x \cos x \sin x}{-\cos x \cos x} = 1$$

$$\text{c. } P = \sin\left(x + \frac{85\pi}{2}\right) + \cos(207\pi + x) + \sin^2(33\pi + x) + \sin^2\left(x - \frac{3\pi}{2}\right)$$

$$= \sin\left(x + \frac{\pi}{2}\right) + (-\cos x) + \sin^2 x + \cos^2 x = \cos x - \cos x + 1 = 1$$

Câu 4: Rút gọn và tính giá trị biểu thức sau:

$$\text{a. } A = \cos(-315^\circ) \cdot \sin 765^\circ$$

$$\text{b. } B = \sin 32^\circ \sin 148^\circ - \sin 302^\circ \sin 122^\circ$$

$$\text{c. } C = \sin 810^\circ \cos 540^\circ + \tan 135^\circ \cot 585^\circ$$

$$\text{d. } D = \sin 825^\circ \cos(-15^\circ) + \cos 75^\circ \sin(-555^\circ)$$

Lời giải

$$\text{a. } A = \cos(-315^\circ) \sin 765^\circ = \cos 315^\circ \sin(720^\circ + 45^\circ)$$

$$= \cos(360^\circ - 45^\circ) \sin(720^\circ + 45^\circ) = \cos 45^\circ \sin 45^\circ = \frac{1}{2}$$

$$\text{b. } B = \sin 32^\circ \sin 148^\circ - \sin 302^\circ \sin 122^\circ$$

$$= \sin 32^\circ \sin(180^\circ - 32^\circ) - \sin(360^\circ - 58^\circ) \sin(180^\circ - 58^\circ)$$

$$= \sin^2 32^\circ + \sin^2 58^\circ = \sin^2 32^\circ + \cos^2 32^\circ = 1$$

$$\text{c. } C = \sin 810^\circ \cos 540^\circ + \tan 135^\circ \cot 585^\circ$$

$$= \sin(720^\circ + 90^\circ) \cos(720^\circ - 180^\circ) + \tan(180^\circ - 45^\circ) \cot(720^\circ - 135^\circ)$$

$$= \sin 90^\circ \cos(-180^\circ) + \tan 45^\circ \cot 135^\circ = -1 + (-1) = -2$$

$$\text{d. } D = \sin 825^\circ \cos(-15^\circ) + \cos 75^\circ \sin(-555^\circ)$$

$$= \sin(720^\circ + 90^\circ + 15^\circ) \cos 15^\circ + \cos(90^\circ - 15^\circ) \sin(-720^\circ + 180^\circ - 15^\circ)$$



$$= \sin(90^\circ + 15^\circ) \cos 15^\circ + \cos(90^\circ - 15^\circ) \sin(180^\circ - 15^\circ)$$

$$= \cos 15^\circ \cos 15^\circ + \sin 15^\circ (-\sin 15^\circ) = \cos^2 15^\circ + \sin^2 15^\circ = 1.$$

Câu 5: Rút gọn và tính giá trị biểu thức sau:

a. $E = 2 \tan 540^\circ + 2 \cos 1170^\circ + 4 \sin 990^\circ$

b. $F = \frac{\sin(-234^\circ) - \cos 216^\circ}{\sin 144^\circ - \cos 126^\circ} \cdot \tan 36^\circ$

c. $G = \frac{\cos(-234^\circ) - \cos 666^\circ}{\sin 1206^\circ + \cos 36^\circ} \cdot \cot 36^\circ$

d. $H = \frac{\sin(-328^\circ) \cdot \sin 958^\circ}{\cot 572^\circ} - \frac{\cos(-508^\circ) \cdot \cos(-1022^\circ)}{\tan(-212^\circ)}$

Lời giải.

a.

$$E = 2 \tan 540^\circ + 2 \cos 1170^\circ + 4 \sin 990^\circ$$

$$= 2 \tan(3 \cdot 180^\circ) + 2 \cos(90^\circ + 3 \cdot 360^\circ) + 4 \sin(-90^\circ + 3 \cdot 360^\circ) = 4$$

b.

$$F = \frac{\sin(-234^\circ) - \cos 216^\circ}{\sin 144^\circ - \cos 126^\circ} \cdot \tan 36^\circ = \frac{\cos(90^\circ + 234^\circ) - \cos 216^\circ}{\cos(90^\circ - 144^\circ) - \cos 126^\circ} \cdot \tan 36^\circ$$

$$= \frac{\cos 324^\circ - \cos 216^\circ}{\cos(-54^\circ) - \cos 126^\circ} \cdot \tan 36^\circ = \frac{-2 \sin 270^\circ \cdot \sin 54^\circ}{-2 \sin 90^\circ \cdot \sin(-36^\circ)} \cdot \frac{\sin 36^\circ}{\cos 36^\circ}$$

$$= \frac{-2 \cdot (-1) \cdot \cos(90^\circ - 54^\circ)}{-2 \cdot 1 \cdot (-\sin 36^\circ)} \cdot \frac{\sin 36^\circ}{\cos 36^\circ} = 1$$

c.

$$G = \frac{\cos(-234^\circ) - \cos 666^\circ}{\sin 1206^\circ + \cos 36^\circ} \cdot \cot 36^\circ = \frac{\cos(126^\circ - 360^\circ) - \cos(-54^\circ + 2 \cdot 360^\circ)}{\sin(126^\circ + 3 \cdot 360^\circ) + \cos 36^\circ} \cdot \cot 36^\circ$$

$$= \frac{\cos 126^\circ - \cos(-54^\circ)}{\sin 126^\circ + \cos 36^\circ} \cdot \cot 36^\circ = \frac{\sin(90^\circ - 126^\circ) - \sin(90^\circ - 54^\circ)}{\cos(90^\circ - 126^\circ) + \cos 36^\circ} \cdot \frac{\cos 36^\circ}{\sin 36^\circ}$$

$$= \frac{\sin(-36^\circ) - \sin 36^\circ}{\cos(-36^\circ) + \cos 36^\circ} \cdot \frac{\cos 36^\circ}{\sin 36^\circ}$$

$$= \frac{-2 \sin 36^\circ}{2 \cos 36^\circ} \cdot \frac{\cos 36^\circ}{\sin 36^\circ} = -1$$

d.

$$H = \frac{\sin(-328^\circ) \cdot \sin 958^\circ}{\cot 572^\circ} - \frac{\cos(-508^\circ) \cdot \cos(-1022^\circ)}{\tan(-212^\circ)}$$

$$= \frac{\sin(32^\circ - 360^\circ) \cdot \sin(238^\circ + 2 \cdot 360^\circ)}{\cot(32^\circ + 3 \cdot 180^\circ)} - \frac{\cos(-148^\circ - 360^\circ) \cdot \cos(58^\circ - 3 \cdot 360^\circ)}{\tan(-32^\circ - 180^\circ)}$$

$$= \frac{\sin 32^\circ \cdot \sin 238^\circ}{\cot 32^\circ} - \frac{\cos 148^\circ \cdot \cos 58^\circ}{-\tan 32^\circ}$$



$$\begin{aligned}
 &= \sin 32^\circ \cdot \sin(180^\circ - 238^\circ) \cdot \tan 32^\circ + \sin(90^\circ - 148^\circ) \cdot \sin(90^\circ - 58^\circ) \cdot \cot 32^\circ \\
 &= \sin 32^\circ \cdot \sin(-58^\circ) \cdot \frac{\sin 32^\circ}{\cos 32^\circ} + \sin(-58^\circ) \cdot \sin 32^\circ \cdot \frac{\cos 32^\circ}{\sin 32^\circ} \\
 &= \frac{\sin^2 32^\circ \cdot (-\cos 32^\circ)}{\cos 32^\circ} - \frac{\cos(90^\circ - 58^\circ) \cdot \sin 32^\circ \cdot \cos 32^\circ}{\sin 32^\circ} \\
 &= -\sin^2 32^\circ - \cos^2 32^\circ = -1.
 \end{aligned}$$

Câu 6: Rút gọn và tính giá trị biểu thức sau:

a. $I = \frac{\cos(-288^\circ) \cdot \cot 72^\circ}{\tan(-162^\circ) \cdot \sin 108^\circ} - \tan 18^\circ.$

b. $J = 2 \sin(790^\circ + x) + \cos(1260^\circ - x) + \tan(630^\circ + x) \cdot \tan(1260^\circ - x).$

c. $K = \frac{1}{\tan 368^\circ} + \frac{2 \sin 2550^\circ \cdot \cos(-188^\circ)}{2 \cos 638^\circ + \cos 98^\circ}.$

Lời giải.

a. $I = \frac{\cos(-288^\circ) \cdot \cot 72^\circ}{\tan(-162^\circ) \cdot \sin 108^\circ} - \tan 18^\circ.$

$$\begin{aligned}
 I &= \frac{\cos(72^\circ - 360^\circ) \cdot \cot 72^\circ}{\tan(18^\circ - 180^\circ) \cdot \sin(90^\circ + 18^\circ)} - \tan 18^\circ = \frac{\cos 72^\circ \cdot \cot 72^\circ}{\tan 18^\circ \cdot \cos 18^\circ} - \tan 18^\circ \\
 &= \frac{\sin 18^\circ \cdot \tan 18^\circ}{\tan 18^\circ \cdot \cos 18^\circ} - \tan 18^\circ = \tan 18^\circ - \tan 18^\circ = 0.
 \end{aligned}$$

b. $J = 2 \sin(790^\circ + x) + \cos(1260^\circ - x) + \tan(630^\circ + x) \cdot \tan(1260^\circ - x)$

$$\begin{aligned}
 J &= 2 \sin(360^\circ \cdot 2 + 70^\circ + x) + \cos(360^\circ \cdot 3 + 180^\circ - x) + \tan(360^\circ \cdot 2 - 90^\circ + x) \cdot \tan(360^\circ \cdot 3 + 180^\circ - x) \\
 &= 2 \sin(70^\circ + x) - \cos x - \cot x \cdot \tan x = 2 \sin(70^\circ + x) - \cos x - 1.
 \end{aligned}$$

c. $K = \frac{1}{\tan 368^\circ} + \frac{2 \sin 2550^\circ \cdot \cos(-188^\circ)}{2 \cos 638^\circ + \cos 98^\circ}.$

$$\begin{aligned}
 K &= \frac{1}{\tan(360^\circ + 8^\circ)} + \frac{2 \sin(360^\circ \cdot 7 + 30^\circ) \cdot \cos(-180^\circ - 8^\circ)}{2 \cos(360^\circ \cdot 2 - 90^\circ + 8^\circ) + \cos(90^\circ + 8^\circ)} \\
 &= \frac{1}{\tan 8^\circ} + \frac{-2 \sin 30^\circ \cdot \cos 8^\circ}{2 \sin 8^\circ - \sin 8^\circ} = \cot 8^\circ - \frac{\cos 8^\circ}{\sin 8^\circ} = 0.
 \end{aligned}$$

Câu 7: Rút gọn và tính giá trị biểu thức sau:

a. $L = \frac{(\cos 44^\circ + \tan 226^\circ) \cos 406^\circ}{\cos 316^\circ} - \cos 72^\circ \cdot \cot 18^\circ.$



$$b. M = \frac{\tan 46^\circ \cdot \sin 44^\circ + \cot(-136^\circ) \cdot \sin 404^\circ}{\cos 316^\circ} - \tan 36^\circ \cdot \tan 54^\circ.$$

$$c. N = \frac{\sin(-328^\circ) \cdot \sin 958^\circ}{\cos 572^\circ} - \frac{\cos(-508^\circ) \cdot \cos(-1022^\circ)}{\tan(-212^\circ)}.$$

Lời giải.

$$a. L = \frac{(\cos 44^\circ + \tan 226^\circ) \cos 406^\circ}{\cos 316^\circ} - \cos 72^\circ \cdot \cot 18^\circ.$$

$$L = \frac{(\cos 44^\circ + \tan(180^\circ + 90^\circ - 44^\circ)) \cos(360^\circ + 90^\circ - 44^\circ)}{\cos(360^\circ - 44^\circ)} - \cos(90^\circ - 18^\circ) \cdot \cot 18^\circ$$

$$= \frac{(\cos 44^\circ + \cot 44^\circ) \sin 44^\circ}{\cos 44^\circ} - \sin 18^\circ \cdot \cot 18^\circ = \sin 44^\circ + 1 - \cos 18^\circ.$$

$$b. M = \frac{\tan 46^\circ \cdot \sin 44^\circ + \cot(-136^\circ) \cdot \sin 404^\circ}{\cos 316^\circ} - \tan 36^\circ \cdot \tan 54^\circ.$$

$$M = \frac{\tan 46^\circ \cdot \sin(90^\circ - 46^\circ) + \cot(-90^\circ - 46^\circ) \cdot \sin(360^\circ + 90^\circ - 46^\circ)}{\cos(360^\circ + 90^\circ - 46^\circ)} - \tan(90^\circ - 54^\circ) \cdot \tan 54^\circ$$

$$= \frac{\tan 46^\circ \cdot \cos 46^\circ + \tan 46^\circ \cdot \cos 46^\circ}{\sin 46^\circ} - \cot 54^\circ \cdot \tan 54^\circ$$

$$= \frac{\sin 46^\circ + \sin 46^\circ}{\sin 46^\circ} - \cot 54^\circ \cdot \tan 54^\circ = 2 - 1 = 1.$$

$$c. N = \frac{\sin(-328^\circ) \cdot \sin 958^\circ}{\cos 572^\circ} - \frac{\cos(-508^\circ) \cdot \cos(-1022^\circ)}{\tan(-212^\circ)}.$$

$$N = \frac{\sin(32^\circ - 360^\circ) \cdot \sin(180^\circ \cdot 5 + 90^\circ - 32^\circ)}{\cos(180^\circ \cdot 3 + 32^\circ)} - \frac{\cos(32^\circ - 180^\circ \cdot 3) \cos(-32^\circ - 90^\circ - 180^\circ \cdot 5)}{\tan(-180^\circ - 32^\circ)}$$

$$= \frac{-\sin 32^\circ \cdot \cos 32^\circ}{-\cos 32^\circ} - \frac{\cos 32^\circ \cdot \sin 32^\circ}{\tan 32^\circ} = \sin 32^\circ - \cos^2 32^\circ.$$

Câu 8: Tính $D = \cos \frac{\pi}{7} - \cos \frac{2\pi}{7} + \cos \frac{3\pi}{7}$

Lời giải

Ta có: $2 \sin x (\cos x + \cos 3x + \cos 5x) = \sin 2x - \sin 2x + \sin 4x - \sin 4x + \sin 6x = \sin 6x$

Do vậy, với $\sin x \neq 0$, ta được: $\cos x + \cos 3x + \cos 5x = \frac{\sin 6x}{2 \sin x}$



Từ đó, với $x = \frac{\pi}{7}$, ta có: $\cos \frac{\pi}{7} + \cos \frac{3\pi}{7} + \cos \frac{5\pi}{7} = \frac{\sin \frac{6\pi}{7}}{2 \sin \frac{\pi}{7}} = \frac{\sin \left(\pi - \frac{\pi}{7} \right)}{2 \sin \frac{\pi}{7}} = \frac{1}{2}$.

Mặt khác: $\cos \frac{5\pi}{7} = -\cos \frac{2\pi}{7}$. Vậy $D = \cos \frac{\pi}{7} - \cos \frac{2\pi}{7} + \cos \frac{3\pi}{7} = \frac{1}{2}$.

Câu 9: Tính giá trị của biểu thức $\tan 9^\circ - \tan 27^\circ - \tan 63^\circ + \tan 81^\circ$

Lời giải

Ta có $\tan 9^\circ - \tan 27^\circ - \tan 63^\circ + \tan 81^\circ = \tan 9^\circ + \cot 9^\circ - \tan 27^\circ - \cot 27^\circ$
 $= \tan 9^\circ - \tan 27^\circ + \cot 9^\circ - \cot 27^\circ = \frac{-\sin 18^\circ}{\cos 9^\circ \cos 27^\circ} + \frac{\sin 18^\circ}{\sin 9^\circ \sin 27^\circ}$
 $= \sin 18^\circ \left(\frac{\cos 36^\circ}{\sin 9^\circ \sin 27^\circ \cos 9^\circ \cos 27^\circ} \right) = \frac{\sin 18^\circ \cdot \sin 54^\circ}{\frac{1}{4} \sin 18^\circ \cdot \sin 54^\circ} = 4$.

Câu 10: Tính giá trị $\cos 15^\circ \cos 45^\circ \cos 75^\circ$ bằng

Lời giải

Ta có $\cos 15^\circ \cos 45^\circ \cos 75^\circ = \frac{\sqrt{2}}{2} \cdot \frac{1}{2} (\cos 90^\circ + \cos 60^\circ) = \frac{\sqrt{2}}{4} \cdot \frac{1}{2} = \frac{\sqrt{2}}{8}$.

Câu 11: Tính giá trị của biểu thức $\cot 30^\circ + \cot 40^\circ + \cot 50^\circ + \cot 60^\circ$

Lời giải

Ta có $\cot 30^\circ + \cot 40^\circ + \cot 50^\circ + \cot 60^\circ = \frac{\sin 90^\circ}{\sin 30^\circ \cdot \sin 60^\circ} + \frac{\sin 90^\circ}{\sin 40^\circ \cdot \sin 50^\circ} = \frac{2}{\cos 30^\circ} + \frac{2}{\cos 10^\circ}$
 $= 2 \left(\frac{2 \cos 20^\circ \cdot \cos 10^\circ}{\cos 30^\circ \cdot \cos 10^\circ} \right) = \frac{8 \cos 20^\circ}{\sqrt{3}}$.

Câu 12: Tính giá trị của $A = \cos 75^\circ + \sin 105^\circ$.

Lời giải

Ta có $A = \cos 75^\circ + \sin 105^\circ = \cos 75^\circ + \sin 75^\circ = \cos 75^\circ + \cos 15^\circ = 2 \cos 45^\circ \cdot \cos 30^\circ$
 $= 2 \cdot \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} = \frac{\sqrt{6}}{2}$.

Câu 13: Tính giá trị của $F = \frac{\sin \frac{\pi}{9} + \sin \frac{5\pi}{9}}{\cos \frac{\pi}{9} + \cos \frac{5\pi}{9}}$.

Lời giải

Ta có $F = \frac{\sin \frac{\pi}{9} + \sin \frac{5\pi}{9}}{\cos \frac{\pi}{9} + \cos \frac{5\pi}{9}} = \frac{2 \sin \frac{\pi}{3} \cdot \cos \frac{2\pi}{9}}{2 \cdot \cos \frac{\pi}{3} \cdot \cos \frac{2\pi}{9}} = \tan \frac{\pi}{3} = \sqrt{3}$.

Câu 14: Cho $\sin a = -\frac{12}{13}$; $\frac{3\pi}{2} < a < 2\pi$. Tính $\cos \left(\frac{\pi}{3} - a \right)$.

Lời giải



Ta có $\cos^2 \alpha = 1 - \sin^2 \alpha = \frac{25}{169}$ mà $\frac{3\pi}{2} < \alpha < 2\pi \Rightarrow \cos \alpha < 0 \Rightarrow \cos \alpha = -\frac{5}{13}$.

Suy ra $P = \frac{1}{2} \cos \alpha + \frac{\sqrt{3}}{2} \sin \alpha = -\frac{5+12\sqrt{3}}{26}$.

Câu 15: Biểu thức $A = \frac{\sin x + \sin 3x + \sin 5x}{\cos x + \cos 3x + \cos 5x}$ được rút gọn thành:

Lời giải

Ta có $A = \frac{\sin x + \sin 3x + \sin 5x}{\cos x + \cos 3x + \cos 5x} = \frac{\sin x + \sin 5x + \sin 3x}{\cos x + \cos 5x + \cos 3x} = \frac{2 \sin 3x \cos 2x + \sin 3x}{2 \cos 3x \cos 2x + \cos 3x} = \frac{\sin 3x}{\cos 3x}$
 $= \tan 3x$.

Câu 16: Tính $B = \cos 68^\circ \cos 78^\circ + \cos 22^\circ \cos 12^\circ - \cos 10^\circ$.

Lời giải

Ta có $B = \cos 68^\circ \cos 78^\circ + \cos 22^\circ \cos 12^\circ - \cos 10^\circ = \cos 68^\circ \cos 78^\circ + \sin 68^\circ \sin 78^\circ - \cos 10^\circ$
 $= \cos(-10^\circ) - \cos 10^\circ = \cos 10^\circ - \cos 10^\circ = 0$.

